Summations and Mathematical Proof

1. Using proof by induction prove that:
   a) $2 + 5 + 8 + \ldots\ldots + (3n - 1) = \frac{1}{2}n(3n+1)$ for all $n \in \mathbf{N}$.
   b) $6^n - 1$ is divisible by 5 for all $n \in \mathbf{N}$.

2. Prove that $\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2$

3. Find (a) $\sum_{k=11}^{20} 2k+1$ (b) $\sum_{k=1}^{\infty} \frac{1}{2^{k-1}}$

4. Write down expressions for $\sum_{r=1}^{n} r^3 - \left(\sum_{r=1}^{n} r\right)^2$ and $\sum_{r=1}^{n} r^3 + \left(\sum_{r=1}^{n} r\right)^2$ and simplify your answers.